Decimal Numeration and Decimal Coinage. By WILLIAM THOMAS THOMSON, F.R.S.E., F.I.A.

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THE Report of the Select Committee of the House of Commons on the subject of "Decimal Coinage," of 1st August 1853, sets entirely at rest any doubt or question as to the great advantages and facilities which would be afforded by the adoption of a system of decimal numeration and decimal coinage.

That the change will accordingly be made, I feel confident; and as the basis of the new arrangement, as well as the method of carrying it out, are of vast importance to the public at large, and in business generally, I have considered it a fit subject of deliberation for this Institute. It may be said that we should have taken an earlier and more prominent part in originating and promoting a change of system, of the importance of which we had individually, I may safely assume, been long convinced; but I am inclined to think that we have wisely reserved our opinions, and that they will be more valuable in the present stage of the discussion (now that the Report of the Select Committee, and the evidence taken before them, has been published), than they would have been earlier in the day. And as practical actuaries, I hope we may be considered qualified to offer sound opinions and advice on this very important question, seeing that we are accustomed daily to work calculations by decimal fractions, converting and reconverting them to and from the timeworn *libræ, solidi,* and *denarii* of our present accounts, and that we are in daily contact with all classes of men in matters of accounting and finance.

I shall not detain you with any lengthened account of the origin and progress of the decimal system; but it may be instructive to some who have not turned their attention to the matter, to learn concisely what is recorded on the subject.

After various progressive changes, "a mighty stride was made in numerical notation by the Greeks, when they distributed the 24 letters of their alphabet into three classes, corresponding to units, tens, and hundreds. To complete the symbols for all the nine digits, an additional appropriate character was introduced in each class." The Greek notation proceeded directly as far as 999; but there were marks or signs which augmented the values one-thousandfold, ten-thousandfold, and so on. Archimedes, Apollonius, and Ptolemy, made important steps in advance, and the latter

invented and applied the sexagesimal scale. "The arithmetic of the Greeks, thus successively moulded by the ingenuity of their great geometers, had attained a singular degree of perfection, and was capable, notwithstanding its cumbrous structure, of performing operations of very considerable difficulty and magnitude. But those masters of science, rich in their mental resources, overlooked the advantages resulting from a simpler mode of arrangement. They had only to ascend more slowly, and proceed by tens instead of periods of myriads; that is, to retain as numerals no more than the first set of their alphabetic characters, which were already employed with a point or short dash, subscribed to denote thousands. This might seem an easy step in the progress of invention, but the current of ideas had already flowed beyond it." *

The Romans derived their knowledge of numerical notation and "palpable arithmetic" from the Greeks; and a denary scale of numeration, or rather of coinage, it has been stated, on the authority of Pliny, can be traced back to the year 266 B.C., when silver money was first coined in Rome.† "The denarius, the chief silver coin down to the time of Constantine, was so termed from its being equivalent to ten asses or ten lbs. of brass (*deni æris*); although, in consequence of the scarcity of silver after the second Punic war, it was declared equivalent to sixteen asses. Then came the division of the denaria into sestertii (the moneys of account were sestertius and sestertium), one sestertius representing the hundredth part of the ordinary gold coin of Rome, the denarius areus; the latter, however, not having been coined until sixty-two years after the silver denarius (204 years B.C.). The convenience of the sestertius must soon have been discovered; for in all matters of account, the reckoning was by sestertii, notwithstanding that the coin used in making payments was generally the denarius. The sestertius formed the unit of account, and was indifferently expressed as sestertius, sestertius nummus, or simply nummus."‡ One thousand of these made the sestertium, and one thousand sestertia made the denis sestertium.

I am inclined, however, to trace another stream to the fountain head, as more likely to lead to the source from which the true

* Leslie's *Philosophy of Arithmetic.*

† We are told by Pliny, upon the authority of Timaeus, an ancient historian, that till the time of Servius Tullius the Romans had no coined money, but made use of unstamped bars of copper, to purchase whatever they had occasion for.

‡ This statement I have taken from the Report on the subject of Decimal Coinage, by the Edinburgh Chamber of Commerce, in consequence of its succinctness; but the articles 'Denarius' and 'Sestertius,' in any of the Encyclopædias, will give more precise information on the subject to those who desire it.

decimal system, *in conjunction with the principle of local value,* was introduced to Europe.

There is no doubt that the Hindoos have long used it; and it is easy, I am informed, to trace the manner in which our numeral symbols have been derived from those of the Sanscrit, in which language there are distinct names for units, tens, &c., up to what we would call hundreds of thousands of millions of millions. That the Persians and Arabs, by whom it was called "Hindoo science," derived it from the Hindoos, seems equally certain; and the date of their acquiring that knowledge appears to have been between the ninth and twelfth centuries. There are three accounts as to the introduction of digital arithmetic with Arabic numerals into Europe—one, that Gerbert, who was afterwards Pope Sylvester II., found it in Spain among the Moors in the latter part of the tenth century, and introduced it into France; another, that Leonard of Pisa introduced it in 1202, in a work entitled *Liber Abbaci;* and a third, that the *Alonsine Tables,* being constructed principally by Moors at the Court of Alonso, King of Castile, must have been the first instance in which the system appeared.

As regards our present system of pounds, shillings, and pence, it appears to have been introduced into England previously to the invasion of William the Conqueror; but it lays claim also to antiquity, for it is possible, I believe, to trace the origin of such division to the reign of Charlemagne, who introduced the Roman pound to modern Europe—this pound or livre (which consisted of a pound weight of pure silver) being coined into twenty sous or sols, the sou being again divided into twelve deniers, which corresponds with our present system.

I shall not, however, detain you with any speculation in these matters of history, curious though they be, but step at once to the period when the Arabic symbols displaced the Roman symbols from accounts in this country, and for the first time introduced a decimal mode of numeration, distinguished from the Greek and Roman systems by the system of "local value," which characterizes the Arabic.

It has been remarked by Peacock, whose valuable history of arithmetic most of you have probably seen or heard of, that the natural scales of numeration alone can and must prevail, and to prove this I feel almost tempted to digress for the purpose of explaining and considering the effect of the various scales as compared with the denary or decimal; but I must confine myself strictly now to the decimal system.*

* The article 'Arithmetic,' in the supplement to the *Encyclopædia Britannica,* contains a very full analysis of these different scales.

The following observations on the value of the Arabic numeral and its decimal arrangements, express very concisely the importance of the system.

"We find ourselves in possession of a method of representing numbers so simple and powerful, that the principle and practice of the most complicated rules follow from it with ease. It is so well known that we need not explain it; but when we separate from the rest the part which particularly distinguishes our numeration from that of the ancient Europeans, we shall find that our superiority consists in the following:—*

"1*st.* The value of a figure depends not only upon the simple numbers for which it stands when alone, but on the *place* in which it stands. Thus, in 888, the three eights mean, eight, eight tens, and eight hundreds.

"2*d.* The place of a figure, considered as affecting its value, is determined by the column in which it stands; and in the absence of succeeding figures to indicate the existence of other columns, their place is supplied by cyphers, which of themselves are considered as having no value. Thus, the value of 8 in 800 is the same as the 8 in 863."

The use of Arabic numerals certainly preceded the invention of printing; but on referring to the works of the early printers, 400 years ago, we do not find the use of the Arabic numerals except occasionally in dates, or such exceptional instances in which we at the present day use the Roman numerals. In the works of Caxton, for example, I am informed that they only appear in a woodcut, till 1480, when he printed the *Mirrour of the World.* Indeed, it was not till the sixteenth century that they came at all into general use. The year "1375," one of the oldest authentic dates in Arabic numerals, is written by Petrarch in a copy of St. Augustin. In 1390 a little tract, published in Germany, *De Algorismo*, explains the digital notation and elementary rules of arithmetic. A curious almanack, written on vellum, is preserved in the library of the University of Edinburgh, calculated for 1482, the numerals given in which closely resemble those used by Caxton. The college accounts in the English Universities were kept in Roman numerals till the early part of the sixteenth century. The change in the parish registers did not take place till 1600. In Scotland the oldest date to be met with is 1490, which occurs in the rent-roll of the diocese of St. Andrews.†

As a matter of curiosity, I now lay on the table for inspection the work of Peele, published in 1569, intituled "*The Pathewaye to*

* It can scarcely be said that the principle of local value prevails throughout the Roman system; but it will be remarked that a smaller symbol before a larger one, in numbers less than 100, denotes a subtraction; after it, an addition—IV = 5—1 = 4. VI = 5 + 1 = 6. IX = 10—1 = 9. XI = 10 + 1 = 11.

† See Leslie's *Philosophy of Arithmetic*—a book I cannot too strongly recommend to the student. It is now very scarce, and should be reprinted. The substance of it will be found in the article 'Arithmetic,' in the supplement to the *Encyclopædia Britannica.*

Perfectnes in th' Accomptes of Debitour and Creditour in manner of a Dialogue. Very pleasaunte and proffitable for Marchauntes and all other that mind to frequent the same; once agayne set forth and very much enlarged. By James Peele, Citizen and Salter of London, Clercke of Christes Hospitall, Practizer and Teacher of the same. Imprinted at London in Paules Church-Yarde. By Thomas Purfoote, dwellinge at the signe of the Lucrece, 16 August 1569."

In this work, which I believe was the first book published in this country on book-keeping by double entry, and to which Professor De Morgan refers in his *Arithmetical Books* as not having seen, but which he informs me he afterwards saw and purchased, or recommended to be purchased, for the British Museum—in this curious old book, it will be found that the journal is kept in Roman numerals and the ledger in Arabic numerals, showing the transition state from one to the other, the Roman being retained in the journal in consequence of not requiring to be added.

It is now the proper time to observe that the decimal arrangement of whole numbers was discovered before the system of decimal fractions. We have hitherto treated of the *gradus ascendentes*, and we must now shortly consider the *gradus descendentes*. It seems very extraordinary that this extension of the system, flowing so naturally from the principle of decimal division, should not have been adopted by the Hindoos. They write fractions, I understand, as we do, only omitting the line which separates the numerators and denominators; but they make no use of decimal fractions, except in some peculiar manner, without any peculiarity of notation in approximating to the square roots of numbers.

The Chinese are understood to have employed the descending terms of the denary scale for a very long period; and, as regards other sources, it would appear that decimal fractions were used by Ramus in his *Arithmetic*, written in 1530, and previously by Buckley and Recorde. Simon Stevinus also wrote an express treatise on decimals in 1582, *La Practique d'Arithmetique;* but we may trace the first introduction of them probably to Regiomontanus, who, about the year 1464, transformed the tables of Sines from a sexagesimal to a decimal scale. The sexagesimal scale was invented by Claudius Ptolemæus, about the year of Christ 200.

Since the fifteenth century, then, when the Arabic numeral and system of local value was introduced, the only material addition which has been made to our arithmetical system has been that of

decimal fractions; but their use has been confined to scientific and professional men. As regards improvement in numeration, in the keeping of accounts, and the arrangement of our coinage, we have certainly stood still for upwards of 300 years.

The evidence taken before the Select Committee has now, I am glad to say, introduced the subject of reform to the public mind, with the view of establishing a decimal system of accounts and coinage in this country; and I really hope the time is not far distant when we shall accomplish important changes in our system, and that not only as regards accounts and coinage, but weights and measures also.

The Committee of last session, in summing up the evidence on the subject taken before them, make the following statement:—

"All the witnesses examined by your Committee concur in the opinion that great advantages attach to a decimal system, as compared with the present system of calculation; and the only points on which any difference of opinion was expressed by them relate to the precise basis which should be adopted, and the practical measures to be employed for introducing the decimal system, so as to produce the least amount of temporary inconvenience, and the smallest extent of unwillingness to encounter the change, on the part of the classes who are the most likely to be affected by it.

"With regard to the inconveniences of the existing system, the evidence is clear and decided. That system is shown to entail a vast amount of unnecessary labour and great liability to error, to render accounts needlessly complicated, to confuse questions of foreign exchanges, and to be otherwise inconvenient.

" On the other hand, the concurrent testimony of the various witnesses is to the effect that the adoption of a decimal system would lead to greater accuracy, would simplify accounts, would greatly diminish the labour of calculations (to the extent of one half, and in some cases four fifths, according to Professor De Morgan, who has made the question his especial study); and, by facilitating the comparison between the coinage of this country and other countries that have adopted the decimal system, would tend to the convenience of all those who are engaged in exchange operations, of travellers and others. An important benefit would be derived in several departments of the public service, and in every branch of industry, from the economy of skilled labour which would result from the proposed change, at the same time that the education of the people generally would be much facilitated by the introduction into our schools of a system so directly calculated to render easy the acquirement of arithmetic."

With these strong expressions and opinions before you, which are stated in even stronger language by the witnesses themselves, I shall not dwell on the advantages of the change, but, holding these proved, at once proceed to consider the basis from which a decimal system of accounts and coinage should proceed, and the best method to be pursued in introducing it.

There have been suggested, as far as my observation has extended, eight plans :—

The £1 or Sovereign Unit.	The 4*s.* 2*d.* or Dollar Unit.
The 10*s.* or Ducat Unit.	The 10*d.* or Franc Unit.
The 2*s.* or Florin Unit.	The Penny Unit.
The 1*s.* Unit.	The Farthing Unit.

It will be perceived that the farthing and penny (it is presumed it is not proposed to make a decimal division of the penny) are the only units suggested in connection with which it is possible to proceed upwards from the lowest denomination; but as it is not probable that any person would keep their accounts in farthings or pennies, let us, instead of calling it the farthing unit, call it the £1. 0*s.* 10*d.* unit (that is, 1,000 farthings), and, instead of calling it the penny unit, let us call it the 8*s.* 4*d.* unit (that is, 100 pence). This arrangement will enable us to treat the whole as if they proceeded downwards, and the following table will show the decimal divisions:—

Units.	1st Decimal Division.	2nd Decimal Division.	3rd Decimal Division.
£1	2*s.*	$2\frac{4}{10}d.$	$\frac{6}{25}$ of 1*d.*
10*s.*	1*s.*	$1\frac{2}{10}d.$	$\frac{3}{25}$ of 1*d.*
2*s.*	$2\frac{4}{10}d.$	$\frac{6}{25}$ of 1*d.*	..
1*s.*	$1\frac{2}{10}d.$	$\frac{3}{25}$ of 1*d.*	..
4*s.* 2*d.*	5*d.*	$\frac{1}{2}d.$	..
10*d.*	1*d.*	..	..
1*d.* or 8*s.* 4*d.*	10*d.*	1*d.*	..
Farthing, or £1. 0*s.* 10*d.*	2*s.* 1*d.*	$2\frac{1}{2}d.$	1 farthing.

These plans did not all come before the Committee with support, but they have each of them their supporters otherwise. Let us examine them separately.

THE £1 UNIT.

This unit has the recommendation of the Committee, and among the witnesses who gave evidence it had a large majority in its favour. The Committee thus express themselves :—

"Your Committee have no hesitation in recommending the present pound sterling. Considering that the pound is the present standard, and therefore associated with all our ideas of money value, and that it is the basis on which all our exchange transactions with the whole world rest, it appears to your Committee that any alteration of it would lead to infinite complication and embarrassment in our commercial dealings; in addition to which it fortunately happens, that its retention would afford the means of introducing the decimal system with the minimum of change. Its tenth part already exists in the shape of the florin or 2-shilling piece, while an alteration of four per cent. in the present farthing will serve to convert that

coin into the lowest step of the decimal scale which it is necessary to represent by means of an actual coin, viz., the thousandth part of a pound. To this lowest denomination your Committee propose, in order to mark its relation to the unit of value, to give the name of mill. The addition of a coin to be called a cent, of the value of ten mills, and equal to the hundredth part of the £1, or the tenth part of the florin, would serve to complete the list of coins necessary to represent the moneys of account, which would accordingly be pounds, florins, cents, and mills."

THE 10*s.* OR DUCAT UNIT.

This unit has many testimonials in its favour throughout the evidence, and the cross-questioning generally brings out the admission that its subdivisions are better than the £1 unit. I shall enlarge on its advantages afterwards.

THE 2*s.* OR FLORIN UNIT.

This unit would be no improvement on the £1 unit. £1 would become ten florins, and the cents and mills would descend from the florins. As it assumes a silver monetary standard which all our greatest authorities repudiate in our existing position, however much they might consider such a standard or a mixed standard desirable, I shall not dwell on this proposal. Sir J. Herschel's opinion as to this unit is as follows:—

"Next comes the florin system, which would reckon all in florins and cents of florins. This makes the pound a natural decimal multiple; and so far, good. But it assumes a silver monetary standard; whereas, for good or for evil, for better for worse, we are married to a gold one. I do not mean to say a silver standard would not be better. I believe it would, and I believe a binary standard, half silver, half gold, at the option of either party to insist on, would be better than either; but gold is our standard of value, and we are lashed on to it, and must be carried along with it, toss as it may."

THE 1*s.* UNIT.

This I hold to be the same as the 10*s.* unit, with the same objections as apply to the florin, as regards a silver monetary standard. Sir J. Herschel disposes of this unit in the following manner:—

"Then comes the shilling system. It has no one point to recommend it but its copper dime. The sovereign must be called a 20-shilling piece; the penny must be demonetised; and we are landed in a system having no relation to any other, in Europe or elsewhere."

THE 4*s.* 2*d.* OR DOLLAR UNIT, AND THE 10*d.* OR FRANC UNIT.

These units I shall take together. I consider them highly objectionable, not only for the reasons as to a silver standard

referred to in the case of the florin, but for other reasons.* As I understand it, the proposal for their adoption is founded on some imaginary theory of assimilating the coinage of the world; but although the world may some day have a universal religion, a universal alphabet and language, the same code of laws, and work entirely in concert, I think we had better legislate for ourselves in the meantime, avoiding the creation of confusion at home by the alteration in value of all our higher coins and notions of value by the introduction of such a system.

I do not mean to detract from the value of the French and American systems, to which the adoption of one or other of these units would allow us to assimilate. I have had experience of both to a limited extent when visiting these countries, and found them simple, and I have no doubt that the calculating powers of our transatlantic brethren have been sharpened by the decimal system; but I see no reason for the proposed assimilation.

The franc system would allow you to retain our present penny unaltered in value; but that advantage would be dearly bought, by the utter confusion which would be created in our notions of value by its adoption, as all our accounts would require to be kept in francs. The dollar would allow the retention of the halfpenny, but it is open to the same objection as the franc.

The following question, put to Professor Airey, and his answer, bears upon coinage, as well as weights and measures. It may be in somewhat different degrees, but still the general effect is similar:—

"482. Do you think a uniform system throughout the world would be a great advantage?—Not so much as would at first sight appear. I suppose of the 20,000,000 of people in England, that not 10,000 have anything to do with the weights of foreign countries; and it is far more important to make the relations among themselves certain and convenient."

THE PENNY UNIT.

This is a mere shifting of the proposal for the adoption of the franc unit. In the words of Sir J. Herschel—

"It would give us a franc not very far from the French, and a pound of 200 pence, which was the old Saxon pound of Ethelbert. I took occasion not very long ago to suggest this for a Canadian pound, but it is quite visionary as applied to England."

I admit that, if the penny could be retained unaltered, a great point would be gained; but it is *impracticable.*

* It has been suggested that the 4*s.* 2*d.*, or dollar, might be made a gold coin; but those persons who have seen, and still more, those who have used, the American gold dollar, will agree with me in considering it a most inconvenient coin.—*See* evidence.

THE FARTHING UNIT.

This unit has been supported by one witness (Mr. Headlam), and the Committee have thought it right to allude to it as the system promoting the greatest amount of advantage after the £1, in the following words:—

"The large number of payments which are now expressed in pence would remain unaltered, and a great portion of those daily transactions in which the mass of the population are engaged would be unaffected by the change; but when it is considered that the adoption of that alternative would, by adding ten pence to the value of the present pound, and a half-penny to that of the shilling, necessitate the withdrawal of the whole of the present gold coinage, and nearly the whole of the silver, and involve the alteration of the terms of all contracts and obligations expressed in coin of either of the latter metals, your Committee would not feel themselves warranted in recommending the adoption of such a proposal."

It has been suggested by one ingenious writer on the subject,* that the depreciation of gold, and the increasing value of silver, call for an increase in value of the silver coins, which the farthing unit gives the opportunity of making; but an increase of a half-penny in the shilling or ten pence in the pound may or may not measure the expected depreciation; "for," in the words of the Committee who reported on Mr. Alexander's paper, "as it is quite impossible to predict the extent to which the value of silver may alter, it is equally impossible to say whether, after any assigned period, the intrinsic value of the present shilling would be an exact decimal of the existing gold sovereigns" or of £1. 0*s.* 10*d.* (the proposed "new guinea" of the writer), or of any other assumed gold coin. But farewell to our prospects of obtaining a decimal system, if we mix up with it the difficult questions which a consideration of the currency gives rise to.

One writer has suggested the coinage of a franc piece, and pieces corresponding to its divisions, which, with the existing gold and silver coins, would make up the difference which the adoption of the farthing unit would create in the higher coins; but, in my opinion, that plan would be still more objectionable and confusing than an entire change.

The following is Professor De Morgan's opinion as to the farthing unit, taken from the evidence:—

"730. Another suggestion has been to keep the farthings, but to change the other coins?—That, I think, would give more trouble altogether than the other change; it would give a great deal of trouble in large commercial transactions.

* A paper read before the Royal Society of Arts in Scotland, December, 1853, by Mr. James Alexander.

"731. Would it effect a complete change of the ideas of money?—Yes; it would alter the exchanges—that is, the names under which exchanges are expressed. It would give the commercial world a great deal of trouble, and I do not see that it would be compensated by any advantages. There was a proposal made, when this present silver coinage was introduced (I think in 1816), for a decimal coinage, that we should take the guinea at 252 pence, and convert that into 1000 mils—that is to say, the 252 pence into 250 pence—which was advocated as the least violent change in the copper; but it found no acceptance, and of late years it has never been revived."

I entirely agree with Professor De Morgan.

After these remarks, I shall now limit my further observations to the £1 *unit,* and 10*s. unit;* declaring myself at once an advocate for the 10*s.* unit.

I shall endeavour to give my reasons for this conviction.

To the £1 unit I object, on the ground that it gives coins of account of the most unuseable description. A *florin* is of itself comparatively the price of few articles—it will always continue to be looked on as a double shilling; and if it is to be a money of account under the decimal system, must be recalled, to be *again* re-issued, so as to be marked in a manner intelligible to the people, and divested of its Gothic letters.* I may further observe, that two millions only of these coins were in circulation at the date of the evidence, while there were thirty-seven millions of halfcrowns in the hands of the public, which would require to be called in, in the event of the adoption of the £1 unit. These coins could not be allowed to exist permanently together without confusion of which the Committee were fully aware; but during the transition state, the period of greatest difficulty, they must circulate simultaneously, as the thirty-seven millions of halfcrowns cannot be withdrawn except by slow degrees. This will increase the difficulty of introducing the change; for even now the confusion between these coins, with the small number of florins in circulation, is very great, and I myself have witnessed intelligent persons unable to judge whether a florin or halfcrown had been given them.

The *tenth of a florin,* and hundredth of £1, becomes $2\frac{4}{10}d.$, the proposed cent—too large for a copper coin, and too small for a silver one; for even our present threepenny piece is apt to slip through our fingers. Pieces of a penny and twopence, in copper, were coined in the reign of George III. The latter (two pennies) did not answer their purpose, and were soon discontinued. It has

* The florin has already been twice issued, having been recalled in consequence of the omission of the letters "D. F." The two sets are of different sizes.

been suggested that a coin of mixed metal should be issued; but Mr. Thomas Hankey, junior, and other witnesses, object to such a plan most strongly. Another suggestion is, that a small silver coin with a copper rim should be introduced; but I find that in 1684 tin farthings were coined with a stud of copper in the centre. Half-pennies of the same metals were struck by James II. and William and Mary; but in 1693 the tin was called in, and the copper renewed.

The *tenth of a cent*, or mill, becomes a near approach to our present farthing; but a binary division is here required to give the half-farthing or halfmill. No doubt, the half-farthing is ignored by the shopkeepers who were examined; but I have no doubt they have their reasons for such opinion, while their evidence all proceeds on their past experience—which, in my humble opinion, does not lead to the conclusion that, although a half-farthing may not be wanted at present, it will not be very useful under a decimal system.

' But I would further observe generally, that a binary division is required of all the coins under the £1 unit system, to make them intelligible to the mass in a transition state.

The retention of the £1 in our accounts in all its integrity is considered an equivalent, then, or rather a sufficient compensation, for the confusion which the introduction of the florin, cent, and mill of the above-mentioned values would create; but I cannot imagine that the Government will attempt this. How would it be possible, for instance, in writing down 8·124 (8 pounds, 1 florin, 2 cents, 4 mills), to make it intelligible to a working man, or even one higher in the social scale, that 1 (the first decimal figure) means 2 shillings (for we shall still for some time count in shillings); that 2 (the second decimal figure) means two tenths of two shillings; and that 4 (the third decimal figure) means four hundredths of two shillings, or four thousandths of one pound? It may be replied that the idea of old coins will be dispelled; but I would humbly say, not so, even among the educated; and we must retain the shilling (old solidus) in as far as possible his old place, otherwise we shall get into inextricable confusion. Will not the alteration of pence under a 10*s*. unit produce the same confusion? I am asked; but I answer, no. The shilling is the poor man's measure, "the poor man's unit" (*see* evidence), as well as the rich; and he will soon learn that ten new copper cents make a shilling, while he will be long in discovering the subdivisions of a florin. '

Another unanswerable objection to the £1 unit is that it will force bankers, merchants, and others to adopt a fourth column in

their accounts, or to reject from them the representation of all sums below a cent that is below $2\frac{4}{10}d.$ At present, all values below a penny are denied representation in business books, so that accounts are kept in pounds, shillings, and pence; but under the pound unit accounts must be kept in pounds, florins, cents, and mills, or we must ignore any sum below $2\frac{4}{10}d.$

Without dwelling longer on the £1 unit system, I shall proceed to set forth the advantages of a 10*s.* unit, of which I have been thoroughly convinced throughout.

The 10*s.* unit would enable us to retain the shilling as a coin of account, being the first decimal of the unit. The tenth of a shilling becomes $1\frac{2}{10}d.$—a convenient coin, which need not exceed in size the present rimmed penny, while the tenth of a penny gives very nearly our present half-farthing.

The 10*s.* scale would run thus—the first decimal of the unit always corresponding with the existing shilling:—

$\frac{1}{10}$th of 10*s.*, or ducat—that is,	1 shilling	=	·1
	2 „	=	·2
	3 „	=	·3
	4 „	=	·4
	5 „	=	·5
	6 „	=	·6
	7 „	=	·7
	8 „	=	·8
	9 „	=	·9
	10 „	=	1.

$\frac{1}{10}$th of 1*s.*, or	1 cent	=	$1\frac{2}{10}d.$	= ·01
	2 „	=	$2\frac{4}{10}d.$	= ·02
	3 „	=	$3\frac{6}{10}d.$	= ·03
	4 „	=	$4\frac{8}{10}d.$	= ·04
	5 „	=	$6d.$	= ·05
	6 „	=	$7\frac{2}{10}d.$	= ·06
	7 „	=	$8\frac{4}{10}d.$	= ·07
	8 „	=	$9\frac{6}{10}d.$	= ·08
	9 „	=	$10\frac{8}{10}d.$	= ·09
	10 „	=	$12d.$	= ·10

$\frac{1}{10}$th of $1\frac{2}{10}d.$, or 1 mill, = $\frac{3}{25}$ths of $1d.$, or $\frac{24}{25}$ths of half a farthing	= ·001
2 mills, or $\frac{6}{25}$ths of a penny, and $\frac{24}{25}$ths of a farthing . .	= ·002
3 mills, or $\frac{9}{25}$ths of a penny .	= ·003
4 mills, or $\frac{12}{25}$ths of a penny, or $\frac{24}{25}$ths of a halfpenny . . .	= ·004
5 mills, or $\frac{15}{25}$ths of a penny . .	= ·005
6 mills, or $\frac{18}{25}$ths of a penny . .	= ·006
7 mills, or $\frac{21}{25}$ths of a penny . .	= ·007

8 mills, or $\frac{2}{2}\frac{4}{5}$ths of a penny . . = ·008
9 mills, or $\frac{2}{2}\frac{7}{5}$ths of a penny . . = ·009
10 mills, or cent, $\frac{3}{2}\frac{0}{5}$ths of a penny, = $1\frac{2}{10}$ = ·010

Now observe this scale—the 1*s.* remains as at present up to 9*s.*, and may be kept in a column by itself. 10*s.* make a ducat, or whatever the authorities may choose to call it; rejecting, however, I would suggest, the name 'pound' or 'decimal pound,' as the sovereign will still be called £1 or double ducat. Mark the ease with which the cents and mills under this unit accommodate themselves to our present coinage:—

5 cents make 6*d.*;
2 cents and 5 mills make 3*d.*;

enabling us to let these coins circulate during the change; and, in fact, only rendering it necessary to withdraw the fourpenny piece. Again, 8 mills approach within 1-25th, or 4 per cent., of 1*d.*, and 4 mills, 2 mills, and 1 mill give you nearly our present halfpenny, farthing, and half-farthing.

We now arrive at the point at which the Gordian knot must be cut; for, whether the 10*s.* unit or the £1 unit be adopted, our copper coinage must all be depreciated by 4 per cent. It would thus be necessary to make it understood by proclamation under the ducat or 10*s.* system, that

8 mills make an old penny.
4 „ „ an old halfpenny.
2 „ „ an old farthing.
1 „ „ an old half-farthing.
Also, that 10 mills make a cent, of which 10 go to the shilling.
That 12 old pennies, or 96 mills, with 4 mills or a halfpenny added (100 mills in all), go to the shilling.
That 6 old pennies, or 48 mills, with 2 mills or a farthing added (50 mills in all), go to the sixpence.
That 3 old pennies, or 24 mills, with 1 mill or half a farthing added (25 mills in all), go to the threepence.

If a person should take change for 1*s.*, in the existing copper coins, there will be an apparent gain of a halfpenny; but to a person asking 1*s.* for copper, there would be an apparent loss. This would of course be the same under the £1 system; but I have no doubt the real value of the coins under the change would become apparent to all very soon; and by gradually withdrawing the 6*d.*, 3*d.*, and copper coinage, substituting 5 and $2\frac{1}{2}$ cent pieces, also 1 mill, 2, 4, 8, and 10 mill pieces, the system would be completed, and I have no doubt the transfer would be easy.

It may be replied, that in this way the public will still continue to count in pence, and will not adopt the decimal values; and so

they will for a time; but I beg you to recollect, that the labouring man and the poor man keep no accounts, and when a man is far enough on in the world to keep books he will soon learn the new system, with tables at his hand showing the value in cents and mills of any sum he may receive in the old coinage.

I entertained at one time the same difficulty which Professor De Morgan has done as to the adoption of the words 'cents' and 'mills,' as in a complete decimal system everything must have its tenths, thousandths, &c., which might render it inexpedient to apply these terms to money, as if they belonged to money alone; but after deliberation, and in consideration that the terms 'pence' and 'farthings' must ultimately be given up, and new names adopted, also considering that it is still proposed to retain the word 'pound,' which has heretofore been common to moneys and weights and measures, I have got over the difficulty as far as it presented itself to my mind, and do not now think it will retard or confuse the new system.

The coins under the 10*s.* unit would be as follows:—

Sovereign, or double ducat.
Ducat, or 10*s.* (unit).
Five-shilling pieces, if wanted.
Halfcrown.
Shilling.
Five-cent piece, which, as pence are to be abolished, I would in future call a "silver half," equal in value to an old sixpence.
2½-cent piece, or "silver quarter," equal in value to an old threepenny piece.
1-cent piece, 10 mills.
8 mills (old penny).
4 mills (old halfpenny).
2 mills (old farthing).
1 mill (old half-farthing).

And, as it would be advantageous in stamping coins to have reference to both ends of the system, decimals of a ducat and number of mills, I would therefore stamp each new coin accordingly. For example,

1*s.* would also be 10 cents, or 100 mills.
1 cent would be 100th of a ducat, one tenth of a shilling, or 10 mills; and so on.

I have now to meet the only point of difficulty in the system I advocate, namely, the representation of accounts in ducats of 10*s.*, thus "confusing all our ideas of value," as alleged in the evidence; and I beg it may be remarked that this alleged confusion is the only point of difficulty stated by the witnesses to the adoption of

the 10*s.* unit. The answers by Sir J. Herschel and Professor De Morgan on the subject may be taken as an example of the whole.

Sir J. Herschel's evidence:—

"There is, first, the ducat system, which takes the halfpound as its unit. I call it the ducat system; some speak of royals; some of Victorias; it is no matter, provided only it is not called a pound—for if you call it a pound, all manner of objections apply to it, for which I refer to Mr. Hankey's evidence.

"This has some very taking points. It preserves the shilling as the silver unit—*the poor man's unit*, as it has been called; it requires only doubling to change pounds into ducats. It would admit of a copper coin to represent its tenth part, a copper cent, which is a real advantage.

"On the other hand, it has, in my opinion, fatal objections. It would double the *numerical* announcement of debts, taxes, liabilities of all kinds, rents, and prices; but what is of more real consequence, and is in my mind unanswerable, is that the bulk of our gold circulation cannot possibly consist of 10-shilling pieces. It is impossible to coin enough of them in a given time to meet emergencies. Now the bulk of your gold coinage must consist of your gold unit. It would never do to have the one great element of all our reckonings thinly scattered among larger pieces as our halfsovereigns are now among the sovereigns. It would be, in short, a mere money of account."

Professor De Morgan's evidence:—

"783. Supposing you were to adopt the unit of 10*s.*, and divide it into 1000 parts, would that afford a greater facility in keeping accounts? —Neither more nor less. Arithmetically speaking, it is a matter of no consequence.

"784. Our present accounts being kept in pounds, shillings, pence, and farthings, would it not be an advantage if we could still keep our accounts in pounds, shillings, pence, and mills?—The suggestion being to retain the 10*s.* as our principal coin of account, to be divided into 1000 parts, the tenth part being 1*s.*, the advantage is, that we keep a well known coin, the shilling, as one of our great coins of account, but the disadvantage I have spoken of before, viz., the destruction of all our associations connected with the pound, and the alteration in a coin which is known all over the world."

Now I am inclined to think, that although we adopt a decimal system, it does not follow that all our books and public accounts must be kept in a manner exactly to correspond with the ducat, or that we must express ourselves in ducats. Is it not very easy to call 200 ducats £100, and, even if we keep our books in ducats, to give our results in pounds, at least till we become accustomed to the change? Or, why not keep our accounts as at present, as regards pounds and shillings, changing the pence and farthings into cents and mills—converting the pounds into ducats only when required for use, in calculation or otherwise? Nothing can be simpler—thus:

£	Sh.	Cts.	Mills.		Ducats.	Sh.	Cts.	Mills.
28	14	5	2		57·	4	5	2
16	17	6	4	or	33·	7	6	4
19	9	5	8		38·	9	5	8
65	1	7	2	=	130·	1	7	2*

I think the plan suggested should get over the only difficulty started as to the 10*s.* unit, and which has been made so much of, on the ground of its altering all our notions of existing accounts and values.

I do not see the force of Sir J. Herschel's objection, that "the bulk of our gold circulation cannot possibly consist of 10*s.* pieces." There is no occasion that it should. The sovereign or double ducat may still keep its place, although not itself the unit.

I have little more to add. In the evidence, and in the different pamphlets which have been published, considerable space is taken up with suggestions and discussions as to the effect of making the penny 4 mills or 5 mills under the £1 unit, and 8 mills or 9 mills under the 10*s.* unit, with reference to the loss or gain to the revenue by the change, in connection with postage stamps, receipt stamps, pontages, &c.; but I do not think it necessary to enter on these points—"all rates fixed by Government can be altered by Government;" and as there is at present a net surplus revenue or profit in the Post Office department of nearly one million per annum, the postage question need not surely be one of much difficulty. The stamp question, looking to the recent changes, should also be easily disposed of; and the income tax, at its present rate, might be made 3 per cent., giving a small advantage to the Government. If private interests stand in the way, let a Government Commission settle the loss by compensation in each individual case, taking the amount out of the revenue, or specially assessing the country for it. Surely so great and important a change is worthy of special legislation.

AIDS TO CALCULATION.

In consequence of the reference made by Professor De Morgan and Dr. Bowring in their evidence to different aids to calculation, I have thought it might be interesting to the Institute, before concluding, to receive some further explanation as to the shwan-pan of the Chinese, and other similar instruments. The following description of the "abacus" is taken partly from Hutton's *Mathe-*

* Those who do not wish to introduce mills into their accounts, can reject them as at present, making $1\frac{2}{10}d.$ of present money the lowest represented value, which assimilates very closely to present practice, and gives a practical advantage over the £1 unit.

matical Dictionary, and partly from Leslie's *Philosophy of Arithmetic.*

'ABACUS' (in Arithmetic), an ancient instrument used by most nations for casting up accounts, or performing arithmetical calculations; it is by some derived from the Greek $a\beta a\xi$, which signifies a cupboard or beaufet, perhaps from the similarity of the form of this instrument; and by others it is derived from the Phœnician *abak,* which signifies dust or powder, because it was said that this instrument was sometimes made of a square board or tablet, which was powdered over with fine sand or dust, in which were traced the figures or characters used in making calculations, which could thence be easily defaced, and the abacus refitted for use. But Lucas Paciolus, in the first part of his second distinction, thinks it is a corruption of *Arabicus,* by which he meant their algorism, or the method of numeral computation received from them.

We find this instrument for computation in use, under some variations, with most nations, as the Greeks, Romans, Germans, French, Chinese, &c.

The *Grecian* abacus was an oblong frame, over which were stretched several brass wires, strung with little ivory balls, like the beads of a necklace, by the various arrangements of which all kinds of computations were easily made.

The *Roman* abacus was a little varied from the Grecian, having pins sliding in grooves, instead of strings, or wires and beads. The abacus or *tabula logistica,* with its furniture, is frequently mentioned in the classics.

> "Quo pueri magnis e centurionibus orti,
> Lævo suspensi loculos tabulamque lacerto."
>
> HOR. Sat. I. vi. 73.

For the purpose of elementary education, this table or board was strewed with sand:

> "Nec qui abaco numeros et secto in pulvere metas
> Scit risisse vafer."
>
> PERS. Sat. I. 132.

The sand used was, according to Martianus Capella, of a sea-green colour:

> "Sic abacum perstare jubet sic tegmine glauco
> Pandere pulvereum formarum ductibus æquor."
>
> Lib. vii. De Arithmetica.

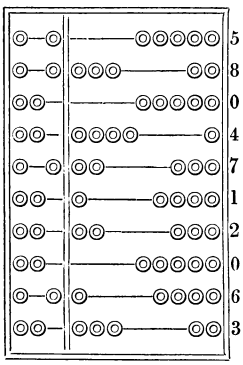
The *Chinese* abacus or shwan-pan, like the Grecian, consists of several series of beads strung on brass wires, stretched from the top to the bottom of the instrument, and divided in the middle by a cross piece from side to side.

In the upper space every string has *two* beads, which are each

counted for 5; and in the lower space every string has 5 beads of different values, the first being counted as *one*, the second as 10, the third as 100, and so on, as with us.

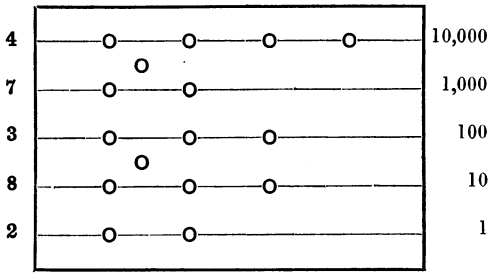
(Mr. Thomson exhibited a Chinese shwan-pan from the museum of the East India Company.)

The Chinese shwan-pan consists of a small oblong board, surrounded by a high ledge, and parted downwards near the left side by a similar ledge. It is then divided horizontally by ten smooth and slender rods of bamboo, on which are strung two small balls of ivory or bone in the narrow compartment, and five such balls in the wider compartment; each of the latter on the several bars denoting *one*, and each of the former, for the sake of expedition, expressing *five*. The progressive bars, descending after the Chinese manner of writing, have their values increased tenfold at each step. The arrangement here figured will hence signify, reckoning downwards, 5,804,712,063. The shwan-pan advances the length of ten billions, and therefore a thousand times farther than the Roman abacus. But the capital improvement which the Chinese had made, was, by commencing the units with any particular bar, to represent the decimal subdivision on the same instrument. Yet this most useful extension of the denary scale, however obvious it may now appear, was unknown in Europe before the time of Stevinus.



The abacus chiefly used in European countries is nearly upon the same principles, though the use of it is here more limited, because of the arbitrary and unequal divisions of money, weights, and measures, which in China are all divided in a tenfold proportion, like our scale of common numbers. This is made by drawing any number of parallel lines, like paper used for music, at such a distance as may be at least equal to twice the diameter of a calculus, or counter. Then the value of these lines, and of the spaces between them, increases from the lowest to the highest, in a tenfold proportion. Thus, counters placed upon the first line signify so many *units* or 1's, on the second line 10's, on the third line 100's, on the fourth line 1000's, and so on; in like manner a counter placed in the first space, between the first and second lines, denotes 5, in the second space 50, in the third space 500, in the fourth space 5000, and so on, so that there are never more than four

counters placed on any line, nor more than one placed in any space, this being of the same value as five counters on the next line below. So the counters on the abacus, in the figure here below, express the number or sum 47,382.



The *Russian* abacus of the present day is called a *shtiot,* and every shopkeeper, every travelling merchant, is provided with one. That modern travellers, who know nothing of the language, find the abacus useful, is fully proved by the following observations, which I find in Oliphant's *Shores of the Black Sea,* lately published (1853):—"Finally, we became very expert in driving a bargain by means of wooden beads strung upon parallel wires, and fastened into a square frame. With these originally-constructed tables, a Russian shopkeeper performs the most elaborate calculations with the greatest rapidity; and though rather perplexing at first, we found them very useful and convenient indicators of sums which it would have been hopeless to attempt expressing in any other way."

Besides the above instruments of computation, there have been several others, invented by different persons—as, Napier's rods or bones, described in his *Rabdologia;* also the abacus rabdologicus, a variation of Napier's, which is described in the first volume of *Machines et Inventions approuvées par l'Académie Royale des Sciences.* An ingenious and general one was also invented by Mr. Gamaliel Somethwest, and is described in the *Philosophical Transactions,* vol. xlvi., where the inventor remarks that computations by it are much quicker and easier than by the pen, are less burdensome to the memory, and can be performed by blind persons, or in the dark as well as in the light. A very comprehensive instrument of this kind was also contrived by the late learned Dr. Nicholas Sanderson, by which he performed very intricate calculations; an account of it is prefixed to the first volume of his algebra, and it is there by the editor called *palpable arithmetic.* Dr. Sanderson had the misfortune to be blind from the age of one year, and contrived

the instrument for his own particular case. By reference to Leslie and Hutton, additional information as to these "aids to calculation" will be received.

I shall only here refer further to the hints thrown out by Professor De Morgan in his evidence as to the extended use of logarithms and of the sliding scales now used in some branches of business. The following is taken from his evidence :—

"The arbitration of exchanges is a very complicated consideration with the present system. In 1802 a book was published by a gentleman of the name of Teschemacher, for the arbitration of exchanges by the use of logarithms. It did not mention the word 'logarithms,' because that would have frightened the mercantile men. It did not come into use, because it was not properly introduced, and was sold at a very high price; and I only mention it as showing my own knowledge that a system of logarithms would be a very material aid in the question of the arbitration of exchanges. I will mention another instance, and that is the sliding rule which the carpenters and engineers use, which is the logarithmic rule, very easy to understand, and easy to apply to any decimal system. It is not impossible to apply a sliding rule of calculation to our system as it is, but though it is not impossible, it is practically impossible; it is impracticable, as the difficulty of using it with pounds, shillings, and pence, would be too great. Brokers, for instance, and persons who have to make calculations and adaptations very quickly to find out the proper price of one stock according to the price of another stock, might use the sliding rule to very great advantage."

If we adopt a decimal system of numeration and coinage, I have no doubt business men will soon become adepts in the use of all these important aids to calculation, and logarithmic tables will come to be used with as much facility as an ordinary interest table.

On Multiplication by aid of a Table of Single Entry.

By J. J. Sylvester, M.A., F.R.S.

A REMARK has been conveyed to me by Mr. Cayley, as occurring somewhere in Gergonne's *Annales*, which is, I think, of sufficient interest for the purposes of practical computation, and especially as applicable to questions of the class which occur in actuarial practice, to justify its introduction to the notice of the readers of this *Journal*. This remark is to the effect that a table of single entry, sufficiently extensive, will serve to give the product of any two numbers by the aid of the processes of addition and subtraction alone, just as is the case in logarithmic computation, but with the advantage over that method of perfect precision in the